Analysis of the Properties of Superconducting Spiral Resonators

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Introduction

- Nuclear Magnetic Resonance (NMR) is the phenomenon in which atomic nuclei from superconducting materials (known as resonators) absorb and re-emit electromagnetic energy (resonate) at a specific frequency (known as resonance frequency).
  - Applications of NMR include the study of various materials, and magnetic resonance imaging (MRI).
- Superconducting spiral resonators (see Figure 1), are used to stimulate and detect the NMR signal. This is useful for high-sensitivity radio frequency (RF) probes.
- Designing RF probes requires an understanding of the properties of spiral resonators.
- To test these properties, the resonators are placed between two loop antennas connected to a network analyzer to form a circuit (see Figure 1).

Purpose and Methods

- This analysis is to gain a better understanding of how these resonators (spiral in shape) behave so more versatile designs can be created.
- By mathematically calculating parameters of this circuit (See Figure 2)—such as impedance and transmission—a resonance frequency known as the fundamental resonance frequency was identified.
- Analysis of this resonance frequency involved the non-harmonic behaviors of spiral resonators that are observed by experiment, but not predicted by the model of transmission line resonators (See Figure 4).

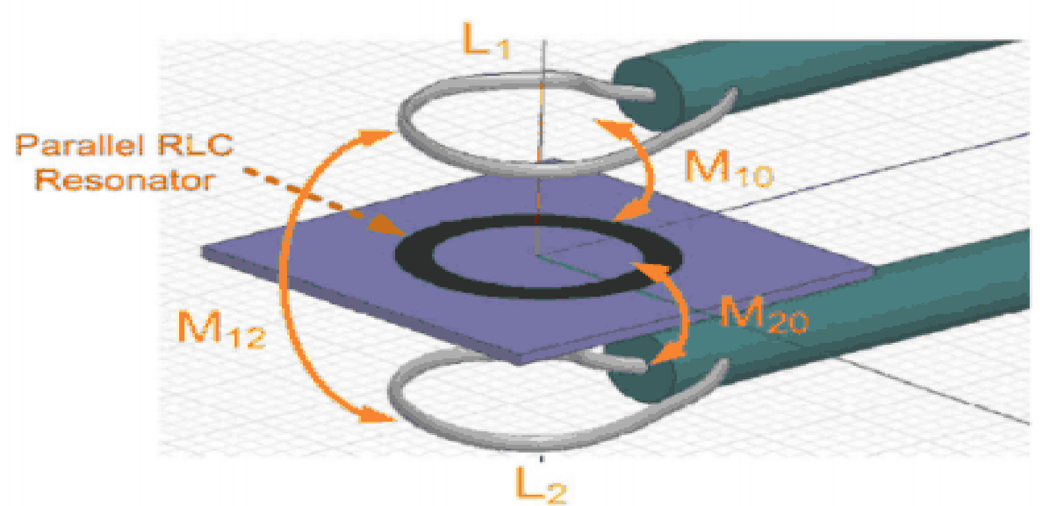


Figure 1: Diagram of the coupling of a spiral resonator between two loop antennas. Emphasis is on the mutual inductances of the two antennas with each other and the resonator. (Reproduced from source [1])

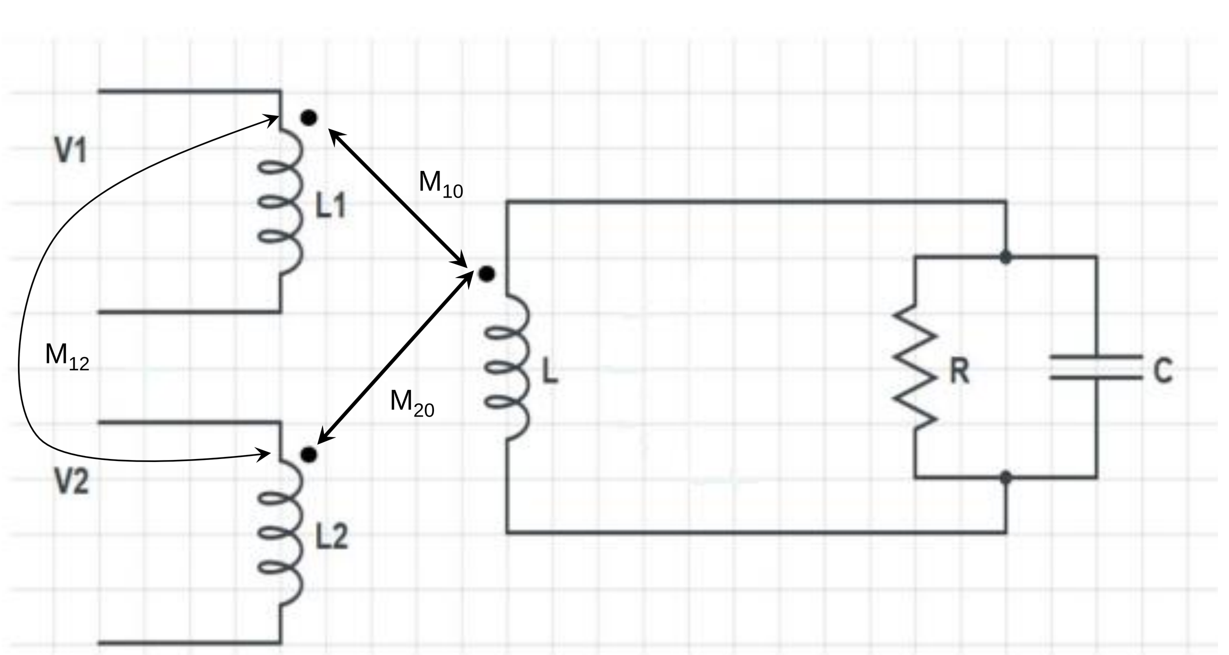


Figure 2: Circuit diagram of the coupling of a spiral resonator between two loop antennas. Resonator is modeled as a parallel RLC circuit . All inductors in this circuit (L, L1, and L2) interact with each other through mutual inductances (M_{10}, M_{20}, and M_{12})

Observations

- Mathematical calculations using impedance and transmission (characteristics of the resonator circuits) allowed for the determination of the fundamental resonance frequency both graphically and mathematically (as seen below).
- We identified the impedance (represented by Z-parameters) and transmission (represented by S-parameters), which described the internal behavior of a circuit network (such as the ones in Figure 2)
  - Z-parameters depend on resistance, inductance, and capacitance.
  - Also influenced by the mutual inductances.
  - They are arranged in a matrix based on the source and receiving antennas (See below).

The matrix of the Z parameters of the circuit found in Figure 1 and Figure 2. This matrix shows how resistance, capacitance, and inductance relate to each other. It shows how mutual inductances between the two antennas can contribute to impedance in the circuit. (Reproduced from source [1])

$$[Z] = \begin{bmatrix} j\omega L_1 + \frac{M_{10}{}^2\omega^2}{Z_{RLC}} & j\omega M_{12} + \frac{M_{10}M_{20}\omega^2}{Z_{RLC}} \\ j\omega M_{12} + \frac{M_{10}M_{20}\omega^2}{Z_{RLC}} & j\omega L_2 + \frac{M_{20}{}^2\omega^2}{Z_{RLC}} \end{bmatrix}$$

The following equations (once the resistance is ignored) illustrate how the resonance frequency can be predicted mathematically. Using parameters C = 17.6pF, L = 500nH, $M_{10} = M_{20}$ = 3.2nH, M_{12} = 190pH.

$$Z_{RLC} = j\omega L + \frac{R}{(1 + j\omega RC)} \xrightarrow{R \to \infty} Z_{RLC} = \frac{1 - \omega^2 LC}{j\omega C}$$

$$Z_{21} = j\omega M_{12} + \frac{M_{10}M_{20}\omega^2}{Z_{RLC}} \xrightarrow{Im\{Z_{21}\} = 0} \omega = \frac{1}{\sqrt{C\left(\frac{M_{12}L - M_{10}M_{20}}{M_{12}}\right)}}$$

$$f = 53.6MHz$$

- S-Parameters are defined by how much power is transmitted from the source antenna to the receiving antenna versus how much power is reflected back to the source.
- Peaks in transmission occur at resonance frequency (as seen in Figure 3).
- Using these peaks, we can find the fundamental resonance frequency and see it match up to calculations with Z-parameters (See Figure 3).

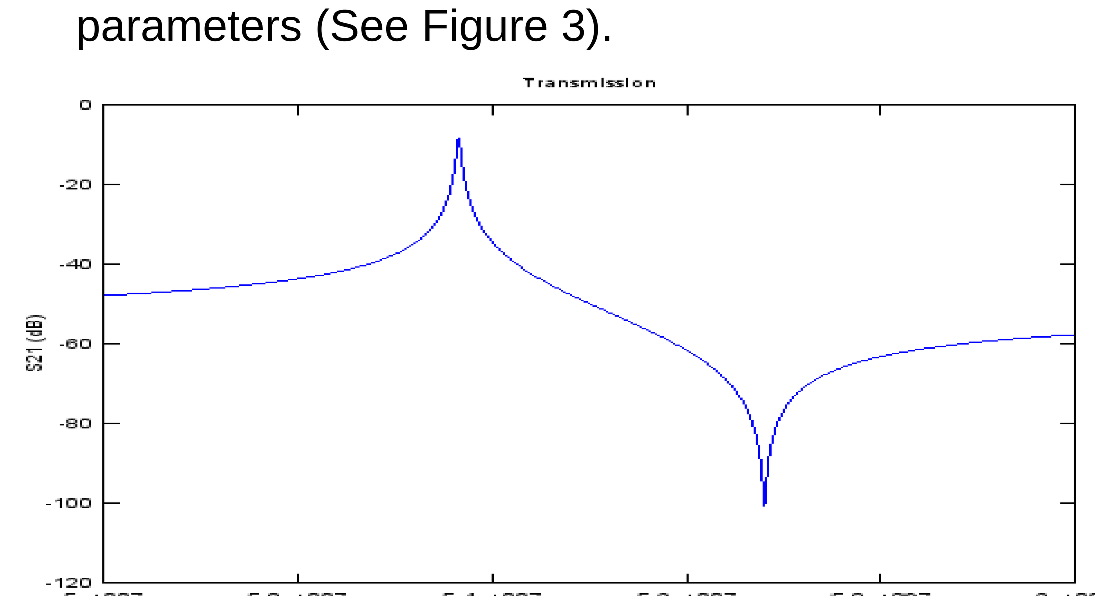


Figure 3: Graph of how energy is transferred from one antenna to the other. At the frequency of 53.6 MHz (the resonance frequency shown by the peak in the graph and calculated in Figure 4), the most energy is transferred from antenna 1 to antenna 2. (Simulated on MATLAB)

- Non-Harmonic Behavior
  - Transmission line resonators have one fundamental resonance frequency, and all other resonance frequencies (known as modes) are integer multiples of that frequency.
  - Spiral resonators have non-harmonic behavior between the first mode and all following modes.
  - Knowing this is important because we can change the higher-mode frequencies at which a coil resonates leading to more versatility in design.

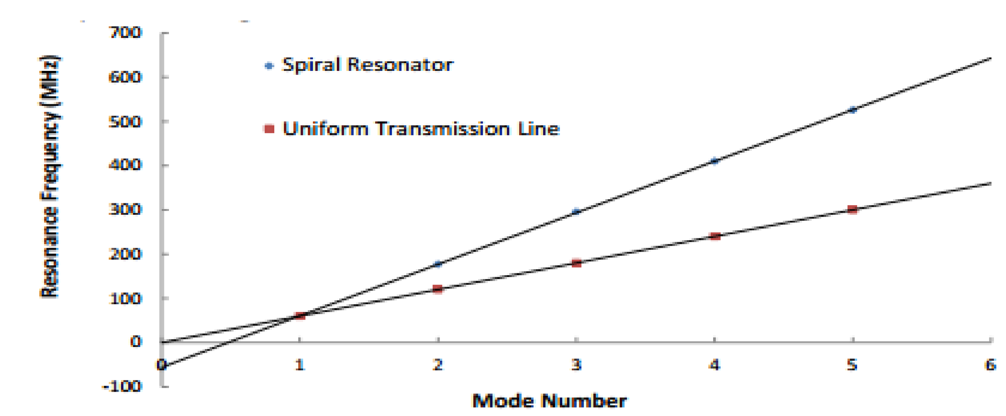
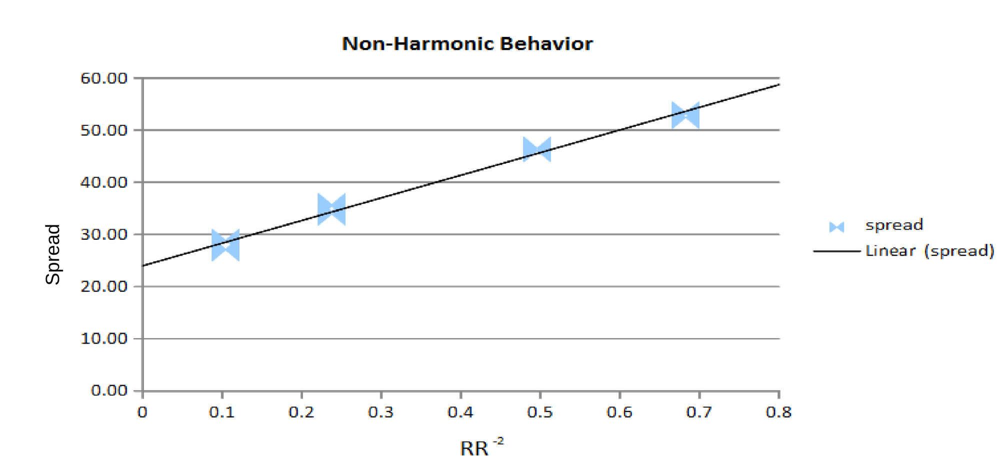


Figure 4: Transmission line resonators have an even spread between modes. Spiral resonators, on the other hand, have an uneven, yet still linear, spread between its first mode and subsequent modes. This contributes to the characteristic non-harmonic behavior of spiral resonators.

- Various properties of spiral resonators were analyzed to determine which most greatly influenced this non-harmonic behavior. The change was inversely linked to the square of the ratio of the outer radius to the inner radius (abbreviated as RR^{-2}).



$$RR = \frac{R_{outer}}{R_{inner}}$$

$$spread = \frac{y - intercept}{slope}$$

Figure 5: The spread between the first mode and following modes (mathematically defined by the y-intercept divided by the slope of the spiral resonator's graph in Figure 4) has an inverse correlation to the square of the ratio of inner and outer radii (RR^{-2}).

Conclusion

•At a given resonance frequency, the maximum energy is transmitted from one antenna to another (Z-parameters/ impedance and S-parameters/transmission were used to identify these frequencies).

•These frequencies were used to determine non-harmonic behavior that is characteristic of spiral resonators.

•The ratio of the outer radius to the inner radius has the most impact on non-harmonic behavior.

•Using non-harmonic behavior of spiral resonators and what impacts it, one can improve the design of spiral resonators and the RF probes that use them.

References

[1]B.G. Ghamsari, J. Abrahams, S. Remillard, and S.M. Anlage. "High-Temperature Superconducting Spiral Resonator for Metamaterial Applications." IEEE, Vol. 23, No. 3; June 2013

Acknowledgements

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